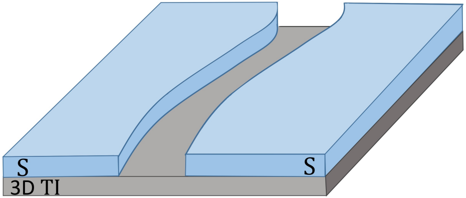
Subgap states and supercurrent in a curved STIS junction



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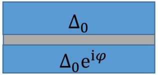
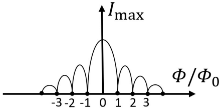
Kazan, Supernano 2025

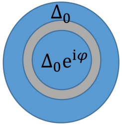
STIS junction



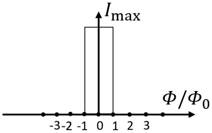
$\Rightarrow$



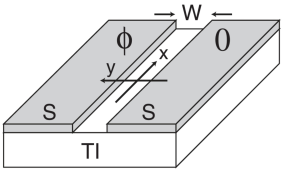
Linear geometry



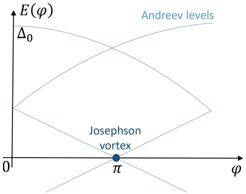
$\Rightarrow$

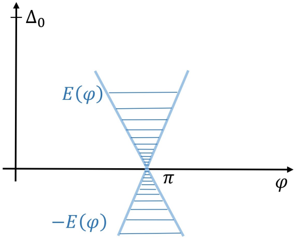


Corbino geometry



[Fu, Kane (2008)]



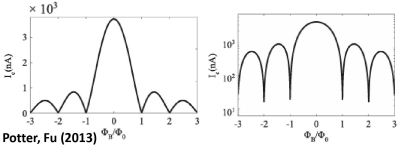


$E_{\pm n} = \pm \hbar\omega\sqrt{n}$

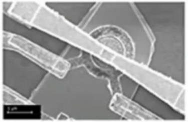
$n = 0$ – Majorana

[Grosfeld, Stern (2011)]

Majorana detection

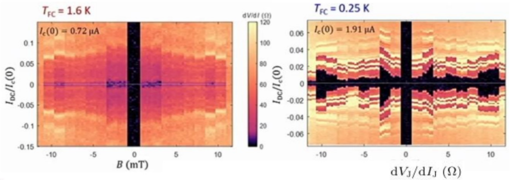


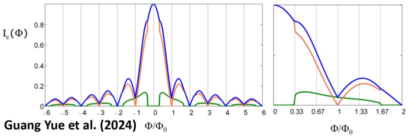
Potter, Fu (2013)



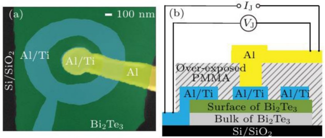
[Park et al. (2024)]

⇒





Guang Yue et al. (2024)



[Zhang et al. (2022)]

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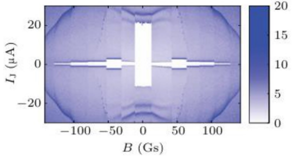
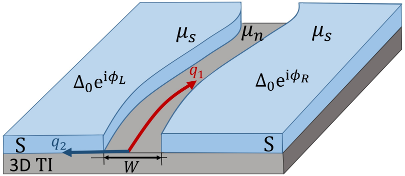


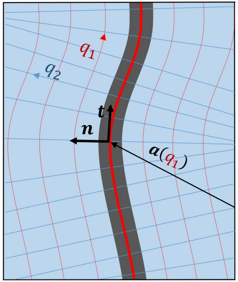
- Maximum current $I_{\max}(\Phi = n\Phi_0) \neq 0$ $\sim$ spatial inhomogeneity
- $I_{\max}$ higher at low temperatures $\sim$ low energy states

Hamiltonian of a curved junction



BdG Hamiltonian $H = v\tau_z \boldsymbol{\sigma}\mathbf{p} - \tau_z \mu(\mathbf{r}) + (\tau^+ \Delta(\mathbf{r}) + h.c.),$

takes rectangular form in coordinates:
$\mathbf{r}(q_1, q_2) = \mathbf{a}(q_1) + \mathbf{n}(q_1) q_2$



Consequences of (q_1, q_2) :

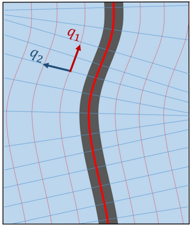
- Problem: the single valued region $|q_2| \cdot \max|\varkappa(q_1)| < 1$, $\varkappa(q_1)$ – local curvature.
  Solution: slightly curved $\varkappa\xi \ll 1$ junction, $\psi \sim \exp(-|q_2|/\xi)$
- Problem: rotate the momentum, but not the spin
  Solution: unitary transform $U_\sigma = e^{-i\sigma_z \theta(q_1)/2}$
- Problem: Jacobian $J = 1 + q_2 \varkappa(q_1)$ in scalar product
  Solution: unitary transform $\tilde{\psi} = \sqrt{J}\psi$, $\tilde{\mathcal{O}} = \sqrt{J}\mathcal{O}\frac{1}{\sqrt{J}}$

BdG Hamiltonian of a curved junction:

$$\tilde{H} = v\tau_z\sigma_x \frac{1}{\sqrt{1+q_2\varkappa(q_1)}}(-i\partial_{q_1})\frac{1}{\sqrt{1+q_2\varkappa(q_1)}} + v\tau_z\sigma_y(-i\partial_{q_2}) - \mu(q_2)\tau_z + (\tau^+\Delta(q_2) + h.c.)$$

Effective low energy Hamiltonian in magnetic field

$$H = v\tau_z\sigma_x \frac{1}{\sqrt{1+q_2\varkappa(q_1)}}(-i\partial_{q_1})\frac{1}{\sqrt{1+q_2\varkappa(q_1)}} + v\tau_z\sigma_y(-i\partial_{q_2}) - \mu(q_2)\tau_z + (\tau^+\Delta_0(q_2)e^{i\phi(q_1)} + h.c.)$$

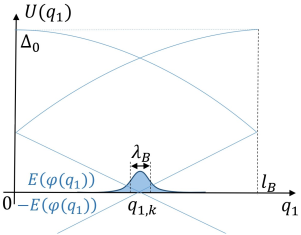


in magnetic field: $\phi(q_1) = \pi + \frac{2\pi}{l_B}(q_1 - q_{1,k})$, $q_{1,k}$ center of k-th Josephson vortex

Averaging over fast blue variable leads to motion in effective potential of Andreev level. Low energy physics correspond to crossing of $\pm E(\varphi(q_1))$ levels.

Effective low energy Hamiltonian in magnetic field:

$$H_{\text{eff}} = E(\varphi(q_1))\rho_z + \frac{1}{2}\rho\left\{\mathbf{v}(q_1),(-i\partial_{q_1})\right\}, \quad v_x \approx v_0 + v_2\varkappa^2(q_{1,k})\xi^2,\ v_y \approx -v_1\varkappa(q_{1,k})\xi$$



$$v_0 = \frac{v\Delta_0}{1+\frac{W}{\pi\xi}}\left[\frac{\sin\left(\frac{W\mu_n}{v}\right)}{\mu_n} + \frac{\Delta_0\cos\left(\frac{W\mu_n}{v}\right) - \mu_s\sin\left(\frac{W\mu_n}{v}\right)}{\Delta_0^2+\mu_s^2}\right]$$

$$v_1 = \frac{\pi v\Delta_0^2}{1+\frac{W}{\pi\xi}}\left[\frac{\frac{W\mu_n}{v}\cos\left(\frac{W\mu_n}{v}\right) - \sin\left(\frac{W\mu_n}{v}\right)}{2\mu_n^2} + i\frac{e^{i\frac{W\mu_n}{v}}\left(1+\frac{W}{\pi\xi} - i\frac{W\mu_s}{v}\right)}{4(\Delta_0 - i\mu_s)^2} - i\frac{e^{-i\frac{W\mu_n}{v}}\left(1+\frac{W}{\pi\xi} + i\frac{W\mu_s}{v}\right)}{4(\Delta_0 + i\mu_s)^2}\right]$$

$$v_2 = \frac{\pi^2 v\Delta_0^3}{1+\frac{W}{\pi\xi}}\left[\frac{e^{iW\frac{\mu_n}{v}}((1-i)+\frac{W}{\pi\xi} - i\frac{W\mu_s}{v})((1+i)+\frac{W}{\pi\xi} - i\frac{W\mu_s}{v})}{8(\Delta_0 - i\mu_s)^3} + \frac{e^{-iW\frac{\mu_n}{v}}((1-i)+\frac{W}{\pi\xi} + i\frac{W\mu_s}{v})((1+i)+\frac{W}{\pi\xi} + i\frac{W\mu_s}{v})}{8(\Delta_0 + i\mu_s)^3} + \frac{2\frac{W\mu_n}{v}\cos\left(\frac{W\mu_n}{v}\right) + \left(\frac{W^2\mu_n^2}{v^2} - 2\right)\sin\left(\frac{W\mu_n}{v}\right)}{4\mu_n^3}\right]$$

Low energy states and Josephson current

Spectrum of low energy states:

$$E_{k,\pm n} \approx \pm\hbar\omega\sqrt{n}\left(1+\frac{v_0 v_2+v_1^2}{4v_0^2}\varkappa^2(q_{1,k})\xi^2\right)\sqrt{\frac{v_0}{v}(1+\frac{W}{\pi\xi})}$$

if $v_0 = 0$: $\quad E_{k,\pm n} \approx \pm\hbar\omega\sqrt{n}|\varkappa(q_{1.k})\xi|\sqrt{\frac{v_1}{v}(1+\frac{W}{\pi\xi})}$

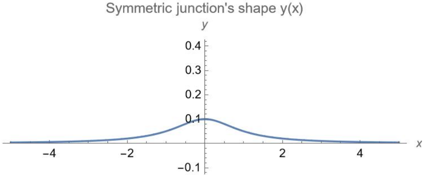
Temperature dependence of current:

$$I(T) = \frac{2\pi}{\Phi_0}\sum_{n,k}\mathrm{n_F}(E_{n,k})\frac{\partial E_{n,k}}{\partial\phi} \quad\approx\quad -\frac{\pi}{\Phi_0}\sum_{n\geqslant 1}\tanh\left(E_n/2T\right)\frac{I_B}{2\pi}\sum_k\frac{\partial E_{n,k}}{\partial q_{1,k}} \quad\propto\quad \sum_k \varkappa(q_{1,k})\varkappa'(q_{1,k})\xi^2 I_B$$

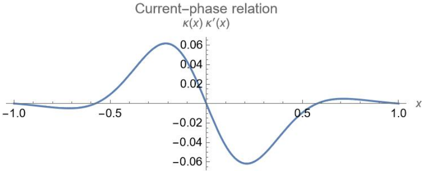
For $\hbar\omega < T \ll \Delta_0$: $\frac{dI}{dT} \approx \frac{9\zeta(3)}{2}\frac{v_0 v_2+v_1^2}{v_0^2}\frac{v/v_0}{1+\frac{W}{\pi\xi}}\frac{\hbar\omega}{\Phi_0}\frac{T^2}{\hbar^3\omega^3}\sum_k \varkappa(q_{1,k})\varkappa'(q_{1,k})\xi^2 I_B$

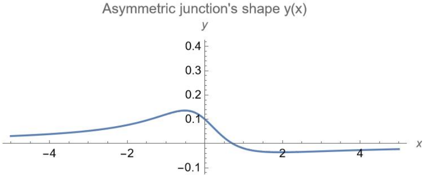
- $I(\Phi = n\Phi_0) \neq 0$
- $I(T)$ higher at low temperatures
- no effect in ideal linear and Corbino geometries
- diode effect is possible

Diode effect

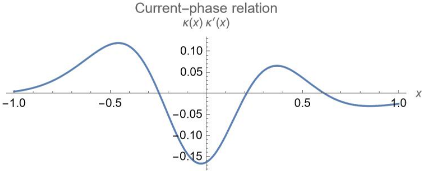


$\Rightarrow$





$\Rightarrow$



No diode effect in case of symmetric junction due to combined symmetry.

$$\Xi = \mathrm{i}\sigma_y \mathcal{I}_x \mathcal{T} \;\Rightarrow\; E(\varphi) = E(-\varphi)$$

	$\mathcal{T}$	$\mathcal{I}_x$	$\mathrm{i}\sigma_y$	$\mathrm{i}\sigma_y \mathcal{I}_x \mathcal{T}$
$\sigma_x p_x$	1	-1	-1	1
$\sigma_y p_y$	1	1	1	1
B	-1	-1	1	1
φ	-1	1	1	-1

Conclusions

1. BdG Hamiltonian of slighty curved $\varkappa\xi \ll 1$ STIS junction
$H = v\tau_z\sigma_x \frac{1}{\sqrt{1+q_2\varkappa(q_1)}}(-i\partial_{q_1})\frac{1}{\sqrt{1+q_2\varkappa(q_1)}} + v\tau_z\sigma_y(-i\partial_{q_2}) - \mu(\mathbf{r})\tau_z + (\tau^+\Delta(\mathbf{r}) + \text{h.c.})$
takes original Dirac form, after projecting on **arbitrary** curve.

2. Curvature induced current contribution:
$I(T) \propto \sum_k \varkappa(q_{1,k})\varkappa'(q_{1,k})\xi^2 I_B$.
   - $I(\Phi = n\Phi_0) \neq 0$
   - $I(T)$ higher at low temperatures
   - Josephson diode effect is possible,
     but absent in case of symmetric junction's shape.

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